The Distribution of Wealth In an Absolutely Conservative Economy in ModEco

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***Abstract:* This paper introduces the ModEco software package, an agent-based evolutionary computer application which can be used to design and study logical model economies on the desktop. It defines the concept of an absolutely conservative ModEco-based economy, an economy in which each commercial transaction leaves both participants with unchanged wealth. It then describes the process by which wealth is distributed in such an economy. Finally, it seeks an explanation for the resulting distribution of wealth in the works published in the new field of econophysics.**

Keywords: agent-based computational economics, finite-state machine, cellular automaton, conservative system, econophysics, evolutionary system, complex adaptive system, wealth distribution, phase space, state space, attractor set, limit set, limit point, economics, Pareto-like tails, fat tails.

I. INTRODUCTION

This paper is the product of a high-school interdisciplinary studies program in which we work with complex adaptive systems. The goal of the study was two-fold: (1) to participate in design discussions as the software was developed, and (2) to build an analytic mathematical economic model of a simple ModEco-based economy. In the pursuit of this goal it became necessary to determine the distribution of the wealth of the agents in the economy, and those efforts have led to these interesting findings. This paper is laid out as follows:

Section II describes the general operating characteristics of the ModEco software. The intended purpose of the software is explained, followed by a discussion of the main operating characteristics. Finally, the positioning of the ModEco software in relation to other known agent-based economic models is discussed.

Section III defines the concept of a *ModEco-based absolutely conservative economic model*. The characteristics of conservative, restorative and sustainable economies are discussed.

Section IV discusses the process whereby a distribution of wealth can be created in an absolutely conservative economy in which all commercial transactions leave both participants with unchanged wealth.

Section V provides a brief review of the history of the Maxwell probability distribution in the development of statistical mechanics. It also briefly reviews the parallel development of the formulae used in the social sciences and in economics when describing the distributions of income and wealth. Finally, it describes how these two paths of development have come together in the work of the econophysicists in the years since 1995.

Section VI presents the results of the application of the concepts of the econophysicists to the data generated by a ModEco-based absolutely conservative, restorative and sustainable economy. The Pareto, Maxwell, Burr and Fisk distributions are matched to the data, and shows that the Burr distribution seems able to model the data flawlessly.

Section VII identifies potential next directions of inquiry for the ModEco project.

Throughout this paper standard references to published papers are noted as numbers in square brackets [1], and web-based references are noted as capital letters in square brackets [A].

II. MODECO OPERATING CHARACTERISTICS

ModEco, very much a work in progress, is conceived as a program in which a student of economics can design an abstracted economy by selecting parameter values, and toggling features on or off. The student can then operate the economy on a desktop computer and watch as the economy develops. Real-time visual output of macro-economic data and micro-economic data allows the student to watch for developing trends. A student can pause the run of such a scenario at any time, examine the details of the economy, and then continue the run. While any single run of an economy in ModEco may appear to be random, it is driven by a pseudo-random number generator seeded under control of the user, so any run is repeatable. The student can also direct the output of macro-economic data and micro-economic data to "comma-separated value" (CSV) files which can be loaded later into a statistical analysis tool such as Microsoft Excel for more detailed study.

"Agent-based Computational Economics" (ACE) [A] is a special interest group of the Society for Computational Economics (SCE) [B]. At the main website of ACE they identify four main strands of research in ACE activities, paraphrased as follows:

1. **Empirical understanding** – Can agent-based models mimic real-world economies?
2. **Normative understanding** – Can agent-based models be used to test the effects of economic policy?
3. **Qualitative insight and theory generation** – Can agent-based models be used to explore the full space of possible economies?
4. **Methodological advancement** – How can we produce better agent-based models?

It is intended that ModEco would be positioned as part of strand number 3. ModEco does not attempt to accurately simulate any real-world economy in which people and firms participate. Rather, it demonstrates abstracted economies, which can be studied in their own right, as logical economies. Therefore no attempt has been made to validate the "correctness" of any economic design, and conclusions drawn from a ModEco-based economy can only be applied to real-world economies with great care. Nevertheless, the goal has been to abstract the general and minimally-necessary characteristics of real-world economies, place them on the student's desktop, and make them observable in detail.

There are many kinds of computer-based economic models. To position ModEco in the field of computational economics, we can look at some taxonomical characteristics. A ModEco-based economy is:

- Agent-based, rather than equation based [C];
- A finite state machine (FSM) [D];
- A cellular automaton [E];
- Evolutionary (i.e. adaptive) with gene-mediated pricing;
- Complete;
- Low Level (i.e. modeling producers and consumers);
- Low in realism; and
- Medium in tractability.

Let's look at these one at a time.

The economy is agent-based: That is to say, logical agents called Prsns and Corps buy and sell goods and services. Corps, as producers, buy raw materiel, hire workers to produce saleable inventories of goods, and sell inventory to consumers. Corps and Prsns, as consumers, buy supplies from producers, consume supplies to meet their daily needs, and sell waste (scrap). A central agent, the materiel manager (MMgr), buys scrap and sells raw materiel. The agents pursue their own self-interest in all actions, and the macro-economic characteristics of the economy emerge from the actions of multiple agents. Daniunas et al. [2] argue that agent-based computing (and computer simulations in general) serves as a third way of doing science, in contrast to both induction and deduction.

There are no economic equations guiding any part of ModEco. While it is possible to measure such macro-economic variables as unemployment rates, price indices, bankruptcy rates, such things are emergent properties of the self-interested behaviour of the agents.

The economy is a finite state machine (FSM): An FSM is a logical construct which can be described in three parts (a) a description of the nature of the space, including the number of dimensions (n) and the extent allowed along each dimension (a finite natural number); (b) which determines a finite number of possible states, expressed as n-tuples; and (c) a mapping (or transition rule) which associates each and every such state (n-tuple) with another target state (n-tuple). The state space of an FSM is most typically a many-dimensional rectangular grid of points (the n-tuples). When the rule is applied, whatever the current state of the FSM, it transitions to the one-and-only new state allowed by the transition rule. The rule is always a one-to-one mapping between n-tuples, or a many-to-one mapping, but never a one-to-many mapping.

A ModEco-based economy is, of course a very complicated FSM with thousands of dimensions and an extent of many millions along each dimension. Such an immensely complicated FSM would be difficult to analyse or study using the tools associated with FSMs. But a ModEco economy does exhibit all of the unavoidable characteristics of an FSM, and so we can at least understand its abilities and limitations in that context.

When we initialize an FSM, we are simply selecting an n-tuple in the state space (also called a phase-space). The action in an FSM proceeds in discrete time (as opposed to continuous time intervals) called ticks of the FSM clock. In one tick, the FSM applies the rule which moves the system from its current state (an n-tuple in the state space) to another state (another n-tuple in the state space). As the rule is applied iteratively, the FSM moves from state to state.

But, since there are only a finite number of possible states, if run long enough (e.g. before the number of ticks exceeds the cardinality of the set of n-tuples) the FSM must, eventually, fall into a cyclic pattern of behaviour. If the cycle has a period of one tick, we say the FSM has a limit point. In a ModEco-based economy such a limit point is commonly achieved. It is the state in which all agents are dead and they stay dead forever. In ModEco we call this the terminal limit point. All unsustainable economies have a trajectory in state space that leads to this terminal limit point. This is to be avoided, if possible. If the cycle has a period greater than one, we say the FSM has a limit cycle. Since a ModEco-based economy has a very, very large number of possible states, such a limit cycle might have a very large period and be impossible to recognise.

There are two necessary characteristics of all FSMs which apply to a ModEco-based economy: (a) given the initial conditions (the initial n-tuple, the initial state of the system) and the transition rule, the trajectory of the FSM through its state space is absolutely and completely pre-determined; and (b) all trajectories of the FSM through its state space must lead either to the terminal limit point (unsustainable economies) or to a limit cycle (sustainable economies).

The economy is a cellular automaton: A cellular automaton is a type of FSM. The action plays out in a rectangular area called the township which is tiled by square cells called lots. As for most cellular automata, the township wraps onto itself at the left and right edges, and at the top and bottom edges. A Corp occupies a commercial lot, or four Prsns may occupy a lot in a single residential complex. Each agent functions within a k-neighbourhood called the commuting area. k is currently set to 2, which means that an agent can enter into transactions with any other suitable agent within a distance of 2 cells (i.e. an area of 25 cells).

The action proceeds in a discrete time unit called a tick of the ModEco clock (the tick of the FSM). During one tick (1) all corps hire up to four workers, if they can, and produce inventory; (2) all unemployed workers with low net worth move to a random cell within their commuting area looking for a better chance for employment; (3) all Corps sell inventory to consumers; (4) all consumers consume supplies, converting them to expertise, health, and scrap (5) all consumers sell scrap to the central MMgr; (6) all Corps buy raw materiel from the MMgr; (7) all agents which are old enough and wealthy enough undergo fission (reproduction with variation); and (8) all agents which are too old, or too poor, or too hungry die without issue, and their assets are transferred to the MMgr. While this 'tick' is quite complicated, it is nevertheless just a transition rule that moves the FSM (the ModEco economy) from one n-tuple in state space to a nearby n-tuple in the state space.

The economy is evolutionary: The action proceeds, tick after tick, and the logical agents pursue their own self-interests in competition with one another for a share of the limited resources. Each agent has a price gene for each type of economic transaction in which it may be involved. For example, Corps have a price gene for buying raw materiel, another for hiring workers, another for selling inventory, and yet another for selling scrap. In every transaction between two agents, each consults its appropriate price gene, and a final price is negotiated. Such prices are called gene-mediated prices, and the transaction is called a gene-mediated transaction.

When an agent becomes old enough and wealthy enough to reproduce, it undergoes fission producing two almost identical daughter agents each having half of the assets of the parent. They differ only in the value of their price genes, which may undergo a small random mutation. Those daughters who receive a beneficial mutation tend to amass greater wealth more quickly than their less-well-adapted sibling. They are therefore more likely to avoid poverty and starvation, and more likely to reproduce and continue their genes in the population of surviving agents. The evolution of the genes causes the average price of the goods and services to drift and this can be viewed in real time as hi-low price charts and price-index charts.

If the price negotiations are based on the intrinsic (and constant) value of the goods and services, then the economy should be stochastically conservative (more on this later) and the price indices should hover near 100. However, if the price negotiations are based on the monetary value of the goods and services, determined by the history of past transactions, serious inflation or deflation is possible.

Such evolutionary computer models are said to be learning or adaptive systems. In the case of ModEco, individual agents learn nothing. They must work with the genes they receive. However, the system as a whole tends to select for those agents with more efficient genes, and so it 'learns' what the most efficient configuration of price genes is for each type of agent.

Randomness: One other point needs to be made in order to understand the capabilities and limitations of a ModEco-based economy. The apparently random elements are not random. A ModEco-based economy is only apparently random. A pseudo-random number generator (PRNG) is used:

- to select agents from a list of agents in random order when seeking to be engaged in a commercial transaction;
- to generate price quotes from the price genes for every commercial transaction;
- to select vacant lots when agents move;
- to generate mutations in the price genes when agents reproduce.

A PRNG is itself an FSM with all of the characteristics of an FSM. The initial condition of the PRNG is called the seed value. Every seed value has an associated trajectory through the PRNG's state space, and an associated limit cycle. The design of a suitable PRNG is a tricky and difficult thing to accomplish. ModEco uses a PRNG called the Mersenne Twister (MT) [F]. The interplay between the limit cycle of the MT PRNG and the transition rule of the ModEco-based economy will determine the period of the limit-cycle of the ModEco-based economy.

Each time a user resets a ModEco-based economy, they are given the option of selecting a seed for the MT PRNG. The selection of a different seed will cause an apparently different random trajectory through ModEco's state space. However, selection of the same seed will cause a precise recreation of the trajectory, since both the initial conditions of the ModEco-based economy are recreated, and the initial conditions of the MT PRNG are also recreated.

The economy is complete: By complete, we mean ModEco models the economy from end to end, from raw materiel to scrap, or, from mine to landfill. A review of the ACE website which publishes a compendium of agent-based economic models would indicate that there are in fact no other agent-based models which attempt to model the complete economic cycle. It appears that ModEco is unique in this respect.

The economy is low level: If we define levels of economic activity based on distance from production, we have (a) low level – producers and consumers; (b) medium level – commodity traders; and (c) high level – traders in stocks, bonds and derivatives. ModEco is intended to be low level.

Low Realism, Medium Tractability: M Christelli et al. [10] review eight representative agent-based models and place them on a two-dimensional grid of realism versus tractability. ModEco, since it is intended to abstract the minimal necessary structure of a working economy, would probably be placed on the lower edge representing with low realism medium tractability. This is relatively far from the corner in which we find the 'ideal model'.

III. DEFINITION OF ABSOLUTELY CONSERVATIVE ECONOMY

In this paper we look at the distribution of wealth in an absolutely conservative economy. The phrase "absolutely conservative" is special to ModEco, and is defined in this section.

ModEco was inspired by the book "The Ecology of Commerce" written by Paul Hawken [1]. The goal in the design of ModEco is to create a model economy which is configurable as:

- A conservative economy;
- A restorative economy; and
- A sustainable economy.

A Conservative Economy – The economy as a whole, at the level of macro-economics, conserves key quantities, in the sense that quantities such as gold, carbon, mass or energy are conserved in real-world physical and economic systems. This is the first of three meanings of ‘conservative’ playing a role in ModEco-based economies. Since few (if any) economic models attempt to model an entire economy from end-to-end (from mine to landfill) they do not consider such macro-level conservation of key quantities to be relevant. Lack of conservation, however, implies unlimited supplies of resources. In ModEco, all key resources are in limited supply, and conserved. This is, arguably, an artifice which does not accurately represent reality. Ok. In ModEco the conserved quantities are (a) cash, (b) intrinsic value, (c) resource-based materiel units, and (d) work-based materiel units. These four types of quantities are conserved at the highest level in all economies in ModEco with one minor exception. The exception arises with cash. There is a toggle called “quantitative easing” which, when ‘on’, enables the central authority to go into debt. Overall the sum of all cash in the economy is nevertheless conserved, but cash may flow into the private sector (Corps and Prsns) endlessly.

Consideration of such high-level conservation laws leads us into the new field of econophysics, in which the techniques developed in the study of physical systems involving many active components (e.g. statistical mechanics) can be applied to the study of economic systems. A cursory review of the econophysics literature as posted in the main website of “The Econophysics Forum” [K] reveals an interest in gas-like behaviour of economic agents [e.g. 5, 11, 12, 13, 14, 15, 16], conservation laws [e.g. 5, 11], and entropy laws [e.g. 8], along with many other topics.

A Restorative Economy – The economy must respect the cycles of nature, in the sense that waste materiel must be recycled and re-used. In other words, a restorative economy restores the environment from which resources are extracted, and, by implication, manages all renewable resources enabling a sustainable economy. This goal leads us to application of the concepts and techniques developed in the study of biological and ecological systems to the study of economic systems. Such key concepts as evolution of populations, ecological niches, population dynamics, carbon cycles, and carrying capacity of ecological systems can be used to understand the nature of a restorative economy.

In ModEco a rudimentary type of restorative economy can be achieved by the simple expedient of forcing equality between the pricing of raw materiel and waste materiel. If the cost of waste disposal must be counted into the cost of raw materiel, then the cost of raw materiel must be greater than or equal to the cost of disposal.

A Sustainable Economy – If our current society and way-of-life are going to survive, and, possibly, if the human race is to survive, we must find a way to create a truly sustainable economy. It is abundantly clear that our current western capitalistic economy is not sustainable.

In ModEco, non-sustainable economies follow a trajectory in state space which leads to the terminal limit point. In the language of FSMs, the initial state is in the attractive basin of the limit point. On the other hand, sustainable economies start in the attractive basin of a limit cycle with a period greater than one. The design of ModEco includes a “ModEco Control Panel” in which design parameters can be changed and features (such as quantitative easing) can be turned on or off. Note that these design changes change the layout of the attractive basins in the ModEco state space. Our experience to date is that in most designs, the attractive basin of the terminal limit point includes all states.

Let us return now to the discussion of the phrase “conservative economy” which needs further clarification in the context of sustainable economies. This is the second of three meanings of the word “conservative”. An economy which is dependent on non-renewable resources is, by definition, not sustainable in the very long run. However, an economy which has minimal dependence on non-renewable resources can be said to be effectively sustainable, or almost sustainable, in the long run. Such an economy conserves the non-renewable resources as much as possible, for the use of later generations, and thereby extends the possible duration of time for which it is sustainable.

When we mention a sustainable economy, we mean to imply that it is conservative in both senses of the word defined herein so far.

Absolutely versus Stochastically Conservative: Let’s now turn our attention to the third meaning of the word “conservative” in ModEco-based economies. At the microscopic level at which binary transactions occur between agents, we say an economy is conservative if the net worth of agents is conserved in binary commercial transactions. For the rest of this paper, when we refer to a conservative ModEco-based economy, we mean to use this restricted sense.

In a ModEco-based economy there is something called “intrinsic value” which is associated with cash and with all goods and services. Intrinsic value, measured in dollars, is one of the four conserved macro-economic quantities. The concept of intrinsic value is based on the daily needs of an agent. A Prsn consumes $40 (intrinsic value) worth of supplies each and every tick of the ModEco clock. A Corp consumes $160 (intrinsic value) worth of supplies each and every tick. If the intrinsic value of the current store of supplies for any agent is insufficient for the day’s needs, that agent dies of starvation. The monetary value (i.e. market price) of goods and services may rise and fall as time passes, but the intrinsic value is conserved, and it is the intrinsic value that determines how much consumption satisfies needs.

In ModEco, the net worth of an agent in a conservative ModEco-based economy is defined as the sum of all stores of intrinsic value. This includes both stores of tangible assets (inventory, supplies, scrap, raw materiel and cash) and stores of intangible assets (expertise and health, which imply the ability to do valuable work). Please note that, in a non-conservative economy, net worth is defined as the sum of all stores of monetary value. In a conservative economy, monetary value equals intrinsic value.

By reason of macro-economic conservation, the four conserved quantities are always conserved in all binary commercial transactions. Net worth is not exactly a fifth conserved quantity. What we are really doing is restricting the ways in which intrinsic value is conserved by adding levels of granularity. A conservative ModEco-based economy has three levels of granularity in the conservations of intrinsic value and, by implication, net worth:

- Macro-economic level – the net worth of the economy as a whole;
- Sector level – the net worth of the public sector (the MMgr) and the private sector (Corps and Prsns as a group); and

- Agent level – the net worth of each agent (MMgr, Corps and Prsns).

Definition: A ModEco-based economy is said to be "absolutely conservative" if (a) every binary commercial transaction between agents for goods and services involves a precise exchange of intrinsic values such that the net worth of both agents remains absolutely unchanged; and (b) the central authority (the MMgr) redresses sector-level imbalances as quickly as possible.

Definition: A ModEco-based economy is said to be "stochastically conservative" if (a) on average, all binary commercial transactions between agents for goods and services involve an equal exchange of intrinsic values; and (b) the central authority (the MMgr) redresses sector-level imbalances as quickly as possible.

The need for the role of the MMgr is explained more fully below.

To configure an absolutely conservative economy in ModEco V1.19A, on initiation, all initiating parameters are set to default, except with the following over-ride – price negotiation is toggled off. This toggle resets a number of things to achieve absolute conservation. (1) All price genes are identical, so all agents agree on a price in all transactions. Since this sets the prices of raw materiel and scrap to be equal, such an economy is also restorative. (2) The base of all negotiations for price is the intrinsic value of the good or service, so the price paid is the intrinsic value. (3) The probability of mutation is set to zero, so all succeeding generations of agents will have the same price genes, and will agree on prices, and will base all prices on intrinsic values.

Hypothesis A: It is our hypothesis that to be sustainable a ModEco-based economy must be either absolutely conservative or stochastically conservative.

Discussion of hypothesis A: To date, as of version 1.19A of the software, we have achieved a sustainable absolutely conservative economy which has survived up to 20 million ticks before being user-terminated. Unfortunately, we have not been able to design any stochastically conservative economy which is also sustainable. Furthermore, so far, all ModEco-based economies which are not absolutely conservative suffer from serious inflation, deflation, or unmanageable concentrations of wealth in a few stores, ending at the terminal limit point within 20 thousand ticks. On the bright side, our time, energy and focus in the ModEco project to-date has been on absolutely conservative sustainable economies, and we have no reason to believe that a sustainable stochastically conservative economy is not possible.

IV. GENERATION OF WEALTH DISTRIBUTIONS IN AN ABSOLUTELY CONSERVATIVE ECONOMY

Our focus, for this paper, is on the distribution of wealth in a ModEco-based economy which is absolutely conservative, in the terms described previously.

One might wonder how wealth becomes distributed if every binary commercial transaction results an equal transfer of intrinsic value such that the net worth of agents remains unchanged.

Agents can change their net worth in an absolutely conservative ModEco-based economy in two ways:

- An agent which is old enough and wealthy enough to reproduce will fission, producing two daughter agents, each having half of the assets of the parent, and each having half of the net worth of the parent;
- An agent which is too old, too poor, or too hungry to survive will die, passing its assets to the MMgr (the public-sector central authority) for later distribution to deserving agents Corps and Prsns (the private-sector recipients).

This second point needs further clarification. When an agent dies without issue, the assets (the net worth) are transferred to the MMgr (the central authority) in its role as estate manager. This is a non-gene-mediated transfer of wealth from the private sector (Corps and Prsns) to the public sector (the MMgr) and must be followed by a reciprocating transfer of wealth back to the private sector. However, the receiving agent no longer exists. The estate is therefore distributed to deserving agents, without cost, in support of the economy. A deserving agent is any agent which has successfully negotiated a commercial transaction, but has insufficient resources to fully benefit from the transaction. For example, if a consumer purchases supplies from a Corp but has insufficient cash to purchase a full day's quota, that consumer can apply to the MMgr for a one-time cash grant sufficient to meet quota. If the MMgr has estate funds available, the MMgr will calculate the amount needed to top up the transaction, the grant will be approved and delivered, and thereby the net worth of the receiving agent is increased.

While this scheme is clearly different from the one-to-one gas-like interactions of agents implied in most discussions of wealth distributions in econophysics papers, due to the proxy nature of the MMgr, it logically amounts to the same thing. The process of fission simply reduces the net worth of all agents by a factor of two once every generation. But, when the economy is functioning at carrying capacity, one half of the population will die without issue in each generation, and their assets are transferred across sector boundaries to the MMgr. The processes of estate management (also called municipal grants) ensure that a selection of one-on-one gene-mediated commercial transactions between private-sector agents result in increases in net worth for deserving agents, involving a reciprocal transfer of assets back to the private sector.

Does this meet our definition of absolutely conservative? At the level of binary commercial transactions, all transactions conserve the net worth of all agents, whether private or public sector. When an agent dies, a sector imbalance is created, which the MMgr reduces through grants to private-sector agents.

It is an interesting aspect of this process that deserving agents tend to be those which are short of key resources. The grants tend to be given to the poorer agents. A detailed analysis of this will need to be put in a later paper. But the overall effect would seem to be that the estate management process takes the assets of the poorest of the poor (those who die in poverty and/or old age) and redistributes it back to the poor.

In ModEco, an agent dies if it is too poor. That is, it dies if its net worth is less than a parameter called the "Cash Death Threshold" (or CDT). This parameter is different for Corps and Prsns, so there are two parameters of interest, being the CDT_C for Corps, and the CDT_P for Prsns. Similarly, an agent reproduces only when it is wealthy enough, under the control of the parameters CRT_C and CRT_P – the "Cash Reproductive Thresholds". When building an analytical mathematical model of a ModEco-based economy, it is therefore necessary to be able to estimate the probability that an agent will either perish or reproduce under the cause of a paucity or excess of net worth. If F(x) is a probability distribution for the wealth of a type of agent (Corp or Prsn) then the probability that an agent is poor enough to perish is given by the equation

$$P(perish) = \int_0^{CDT} F(x)dx$$

and the probability that an agent is wealthy enough to reproduce is given by the equation

$$P(reproduce) = \int_{CRT}^{\infty} F(x)dx\,.$$

So, in the quest to construct an analytic mathematical economic model for a ModEco-based economy, a function F(x) is required, and this lead us into econophysics and gas-like models of the distribution of wealth.

V. RECENT INTEREST IN WEALTH DISTRIBTUIONS

The foundations of probability theory were laid by Blaise Pascal and Pierre de Fermat in 17th century. The nineteenth century saw widespread and rapid development in probability theory as it was applied to a wide variety of phenomena. James Clerk Maxwell (1831-1879) in Scotland, Josiah Willard Gibbs (1839-1903) in the USA, and Ludwig Eduard Boltzmann (1844-1906) in Austria applied probability theory to problems in physics and chemistry with great success.

The Maxwell distribution [L] describes the distribution of the speeds of molecules in a container of ideal gas close to thermodynamic equilibrium. One formulation of the equation is

$$P(v) = \sqrt[2]{\frac{2}{\pi}\left(\frac{m}{kT}\right)^3} \bullet v^2 \bullet e^{-\left(mv^2/2kT\right)}$$

where P(v) is the probability that a molecule has speed v; m is the mass of a molecule; k is the Boltzmann constant; and T is the temperature of the gas, measured in degrees Kelvin. There are now several versions of this distribution, under the names Maxwell-Boltzmann distribution, or Boltzmann-Gibbs distribution, which describe the distribution of speed, velocity, momentum or energy in a variety of circumstances.

Boltzmann refined and generalized the concept in his studies of thermodynamics and entropy. The Boltzmann distribution [M] describes the distribution of energy states of a substance which has high enough temperature and low enough density that quantum effects can be ignored. The equation is

$$\frac{N_i}{N} = \frac{s_i \bullet e^{\left(-E_i/kT\right)}}{\sum_i s_i \bullet e^{\left(-E_i/kT\right)}}$$

where N_i/N is the fraction of particles in states i having energy E_i; s_i is the number of states having energy E_i; k is the Boltzmann constant; and T is the temperature of the gas, measured in degrees Kelvin. The Maxwell distribution is a special case of the Boltzmann distribution. This expression is also known as the Gibbs measure. Gibbs is considered the founder of the field of studies called physical chemistry.

These distributions used in the physical sciences have three characteristics which are of particular interest in this discussion:

1. They are theoretical in nature, having been developed from first principles using Newtonian, relativistic or quantum mechanics, having been substantiated by empirical evidence, and being shaped and scaled by physical constants with associated units of measure;
2. They are consistent with universally accepted physical laws such as the conservation laws of energy and mass, and the laws of thermodynamics; and
3. Due to the tight link between theoretical underpinnings and empirical evidence, the number of mathematical formulae is relatively small, and there is common agreement on exactly which formula should be applied to each situation.

Starting late in the 19th century, probability theory was also being applied to social phenomena. The Italian economist Vilfredo Pareto [G, 3, 9] first published a description of the tail of the wealth distribution in which he noted that it can be modeled by a power law $f(x) \sim x^{-(1+\alpha)}$. However, a relative lack of data and an inability to measure social phenomena in a laboratory setting have necessitated a different kind of progress in the study of social phenomena. Commercial needs spurred the exploration of the statistical characteristics of mathematical curves, and the empirical matching of curves to data. For example, the insurance industry, spurred by the need to better understand the statistical characteristics of the populations being insured, made some significant headway in describing populations of events (e.g. deaths) and durations of time prior to events (e.g. lifespans).

In 1942 Irving Wingate Burr (1908-1989) published a compendium of probability distributions [4]. One of these now bears his name. The "Burr Type XII Distribution" is now known simply as the Burr distribution [I] (also called the Singh-Maddala distribution, or the generalized log-logistic distribution; many distributions have several names in the statistical literature). The Burr distribution is often used to model household income for American families. One formulation of the equation is

$$P(x) = c \bullet k \bullet \frac{x^{c-1}}{(1+x)^{k+1}}$$

where P(x) is the probability that a household will have income x; and c and k are shape parameters greater than zero. In mathematical statistics, most curves are in a standard position and have a standard scale. If we add location and scale parameters, the Burr distribution becomes a four parameter distribution. Burr's purpose in publishing his compendium was to give social researchers a selection of distributions which could be used to match data.

A variation of the Burr distribution is called the Fisk distribution [J], which also goes by the name of the log-logistic distribution. It is often used for survival analysis. Given a population of possible future events (e.g. deaths, machine failures, returns to work after major illness) what is the probability that such an event will occur at a given time x. One formulation of the Fisk distribution is

$$P(x) = \frac{\beta}{\alpha} \bullet \frac{\left(x/\alpha\right)^{\beta-1}}{\left(1+\left(x/\alpha\right)^{\beta}\right)^{2}}$$

where alpha is a scale parameter and beta is a shape parameter. The addition of a location parameter would make this a three-parameter curve.

The social sciences have used a number of other distributions to describe wealth, such as the log-normal distribution, the Weibull distribution, the gamma distribution, and the logistic distribution. The selection of a distribution for use in a particular circumstance seems to be largely empirical, in answer to the question "Which distribution best fits the historical data available, and is therefore most likely to be accurately predictive of future events and distributions?" Most distributions are associated with guidelines suggesting circumstances in which they have been successfully used in the past.

These wealth and income distributions used in the social sciences do not have the same standing as those used in the physical sciences for at least three reasons:

1. They are largely mathematical in nature shaped by dimensionless parameters in which units of measure have little role, and measurable physical constants play no role;
2. They are not required to be consistent with any known laws of conservation or behaviour identified in the social sciences; and
3. Due to the lack of widely-accepted theoretical underpinnings, the number of mathematical formulae is relatively large, and the decision respecting which formula should be applied in which situation tends to be a matter of choice guided by custom, experience and rules of thumb based on past successes.

All of these curves mentioned so far (excluding the Pareto distribution) have a characteristic shape, starting at the origin and stretching to the right, the curve rises quickly to a smoothly rounded peak, and then drops along a curve which is monotonically decreasing and is asymptotic to the x axis as x approaches infinity. However, there is one significant difference between the shapes of the distribution formulae of statistical mechanics (including physical chemistry) and the shapes of those commonly used in the social sciences. The curves used in the social sciences, selected for empirical reasons, all have fat right-hand tails. The right-hand tails of these curves follow a type of power-law, and are described as Pareto-like.

Econophysics is a relatively new field of study which is gathering momentum. According to V M Yakovenko [5], the term econophysics was first introduced by the statistical physicist Eugene Stanley at the conference "Dynamics of Complex Systems" in Kolkata (formerly Calcutta) in 1995, and printed in its proceedings in 1996 [6, 7]. In the field of econophysics, those steeped in the statistical techniques and insights of the physical sciences seek to apply those techniques and insights to the resolution of difficult issues in the study of economics.

Since 1995, a great deal of interest has been expressed by econophysicists in the distribution of wealth and income. In large measure, this has been enabled and encouraged by the availability of massive amounts of data associated with tax records placed in the public domain. Around the world physicists who have experience working with the massive data bases associated with high-energy physics experiments are turning their attention and tools and techniques to this economic data. Consistently this economic data has been shown to have "fat tails" (Pareto-like tails), attached to gamma-like or Maxwell-like bodies [5].

When we look for an answer as to why these tails are fat, we start to approach some theoretical underpinning for the selection of the best curve to describe this type of data. In statistical mechanics, mindless ideal-gas molecules may, by one-on-one chance physical encounters, be given an abnormally high share of energy (or speed), but in ensuing collisions with other particles, they will again revert towards the average speed and lose this abnormally excessive energy. Much of the mechanism that distributes wealth in a social system is assumed to be the same; by one-on-one chance commercial encounters resulting in some agents being wealthier than others. However, the agents in a social system are not mindless particles, and those that enjoy wealth are likely to want to avoid dissipating that wealth. In a physical system, particles do not try to avoid collisions that dissipate their energy, but in a social system, agents will try to avoid encounters that dissipate their wealth. So, random chance will ensure that some agents become wealthy, but the social desire to hang onto that wealth will ensure that many will remain in that right-hand part of the tail long after random chance would dictate. This results in fat tails in social distributions for circumstances in which it is desirable to be in the tail, and, presumably, skinny tails in social distributions for circumstances in which it is not.

G Kaniadakis [8] sets a criterion which he uses to further extend the insights of physics into economics. It has been shown that the Maxwell distribution for an ideal gas at moderate temperature can be derived from first principles using the Maximum Entropy Principle. I.e. the Maxwell distribution is the only probability distribution of speeds (and energies) in an ideal gas for which entropy is maximized. Since, by thermodynamic laws, any system, if left alone to age, will reconfigure to maximise entropy, the application of this law makes the Maxwell distribution the only correct distribution for that scenario. In a paper which promises to move the scrimmage line forward significantly, G Kaniadakis:

1. Presents a general four-parameter expression (excluding parameters of location and scale) for a family of mathematical curves that includes a large number of the power-law tailed curves (curves with Pareto-like tails) described in the literature;
2. Applies the "Maximum Entropy Principle" to identify a limited number of sets of parameters which produce curves that are consistent with entropy considerations.

His paper would appear to imply some potential advances in the study of social systems. We know that there are several ways, specific to each type of system under study, to define entropy. In each type of entropy there is a maximum entropy principle. For example, we have thermodynamic entropy in statistical mechanics and Shannon entropy in information theory. We could similarly define a form of entropy for social systems, and use that to deepen our understanding of social phenomena.

In the cited study, Kaniadakis provides an extensive survey of relevant formulae, and an informative bibliography.

VI. DISTRIBUTION OF WEALTH IN AN ABSOLUTELY CONSERVATIVE ECONOMY

When looking at a ModEco-based absolutely conservative economy, we can make three additional hypotheses, each somewhat more difficult to address. This paper will address **Hypothesis B** only, and the others will need to be addressed in a later paper.

Hypothesis B: The Fisk distribution, which has often been often used for income and lifetime analysis, is empirically the best fit for the distribution of net worth in a ModEco-based absolutely conservative economy, based on its historical usage as per M P McLaughlin [9].

Hypothesis C: The distribution of the net worth data is consistent with a maximum entropy principle and can be described using a distribution determined by one of the acceptable sets of parameters identified by G Kaniadakis.

Hypothesis D: It is possible to define a version of entropy for a ModEco-based absolutely conservative economy such that a net worth distribution can be derived from first principles.

Discussion of hypothesis B: An absolutely conservative, restorative and sustainable ModEco-based economy was run for over 45,000 ticks to eliminate any effects of an initial transient trajectory of the FSM. Thereafter, net worth data was collected every 800 ticks (i.e. every generation) for a duration of over 1 million ticks, netting data for 34,530 Corps and 138,801 Prsns. A histogram of 100 bins each was created for both Corps and Prsns.

The data was then moved into a series of Excel spreadsheets which were used to perform a numerical least squares evaluation. The Corp and Prsn data were processed for each of these curve types: Pareto, Maxwell, Burr and Fisk; eight spreadsheets in all.

The fully parameterized formulae of M P McLaughlin were used, as follows:

- **Pareto** – Two parameters, 1 location, 1 scale [G, 9];
- **Maxwell** – Two parameters, 1 location, 1 scale [H, 9];
- **Burr** – Four parameters, 1 location, 1 scale, 2 shape[I, 9];
- **Fisk** – Three parameters, 1 location, 1 scale, 1 shape [J, 9].

Pareto Distribution

$$PDF = B\frac{A^B}{X^{B+1}}$$

A – Location; B – Scale. There are several variations of this formula, all referred to as the Pareto distribution.

Maxwell Distribution

$$PDF = \sqrt{\frac{2}{\pi}}\frac{1}{B}\left(\frac{x-A}{B}\right)^2 e^{\left(-\frac{1}{2}\left(\frac{x-A}{B}\right)^2\right)}$$

A – Location = 0; B – Scale. The Maxwell distribution is a special case of the Chi-square distribution with three degrees of freedom.

Burr Distribution

$$PDF = \frac{C*D}{B}\left(\frac{x-A}{B}\right)^{C-1}\left(1+\left(\frac{x-A}{B}\right)^C\right)^{-D-1}$$

A – Location; B – Scale; C, D – Shape. The Burr distribution with D = 1 becomes the Fisk distribution.

Fisk Distribution

$$PDF = \frac{C}{B}\left(\frac{x-A}{B}\right)^{C-1}\left(1+\left(\frac{x-A}{B}\right)^C\right)^{-2}$$

A – Location; B – Scale; C – Shape. The Fisk distribution is also called the Sing-Maddala distribution, or the log-logistic distribution. It is also a special case in other families of curves.

The results of the least squares exercise in search of a curve of best fit are shown in Table I below.

It would appear that the Fisk distribution has a systematic variation from the collected data, showing the same variation in both cases, curving downward a little too quickly when coming off the peak, but not approaching zero quickly enough for very high net worth. The tail is not fat enough, then too fat.

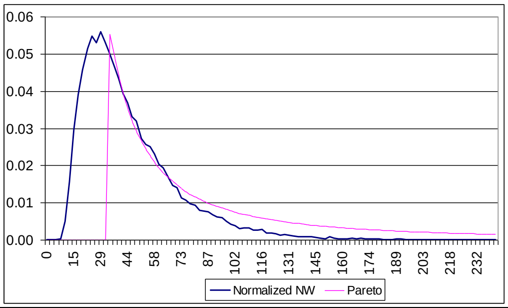
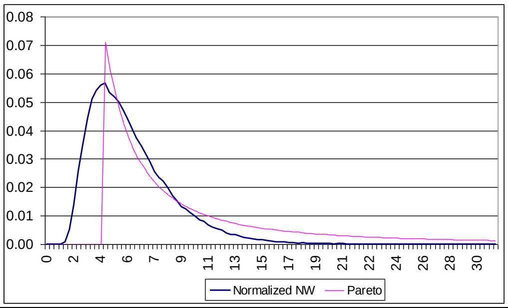
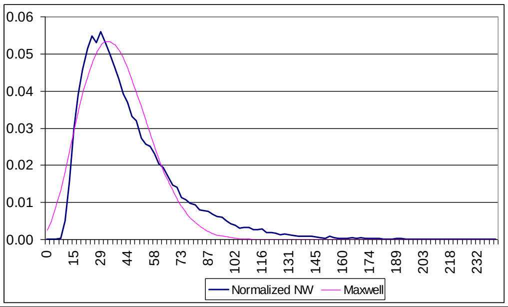
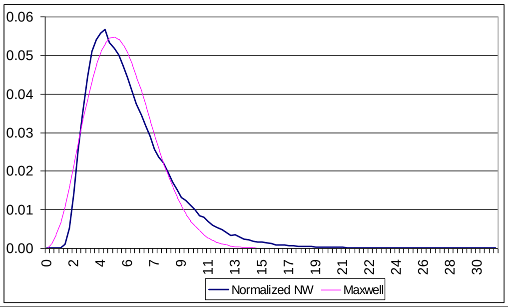
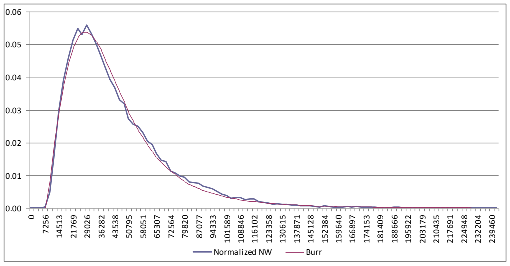
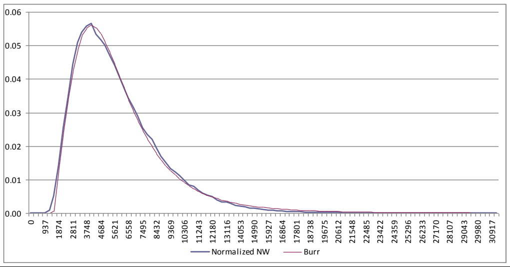
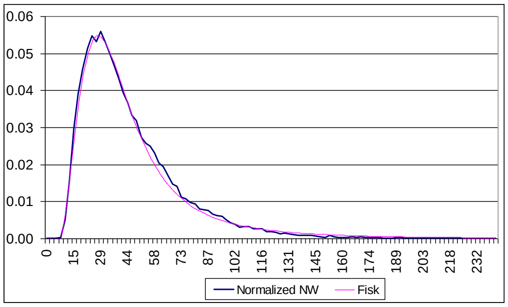
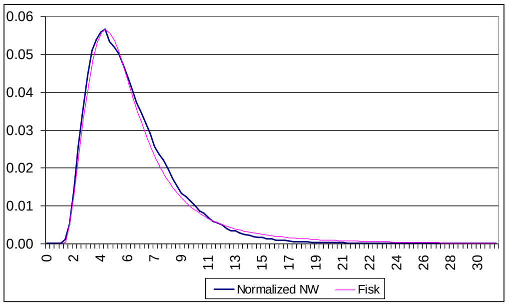
On the other hand, the Burr distribution provides an almost perfect fit. This is not surprising from a mathematical point of view, as it has one additional shape parameter. Our hypothesis B thus appears to have been incorrect. The Burr distribution provides the best empirical fit of those distributions tested so far.

VII. NEXT STEPS IN THE MODECO PROJECT

The future development of the ModEco project has several different possible paths to follow, all of which are intriguing, and some of which may be non-complimentary:

- Further explore hypotheses A, B, C and D above, to gain a deeper understanding of an absolutely conservative ModEco-based economy.
- Refine the ModEco program with the goal of further exploring necessary and sufficient conditions for a restorative and/or sustainable economy in its most abstracted form.
- Refine the ModEco program with the goal of producing a stochastically conservative economy which is sustainable. This is a step towards realism.
- Refine the ModEco program to introduce more realism, such as variations in productivity of agents, variations in intrinsic value of goods and services, explicit energy flows in parallel to the flows of intrinsic value.
- Refine the concept of net worth which, in ModEco, includes the intrinsic value of intangibles such as corporate expertise and personal health and well-being, to include concepts like shelf-life, or economic leverage.
- Add trader agents who buy and sell commodities, stocks and bonds.

Table I – Comparison of Empirical Data for Wealth Distribution and Curves of Best Fit.

CORPS	PRSNS
Pareto 	Pareto 
Maxwell 	Maxwell 
Burr 	Burr 
Fisk 	Fisk 

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